

A Linear Analysis for the Flight Path Control of the Cassini Grand Finale Orbits

Dr. Mar Vaquero

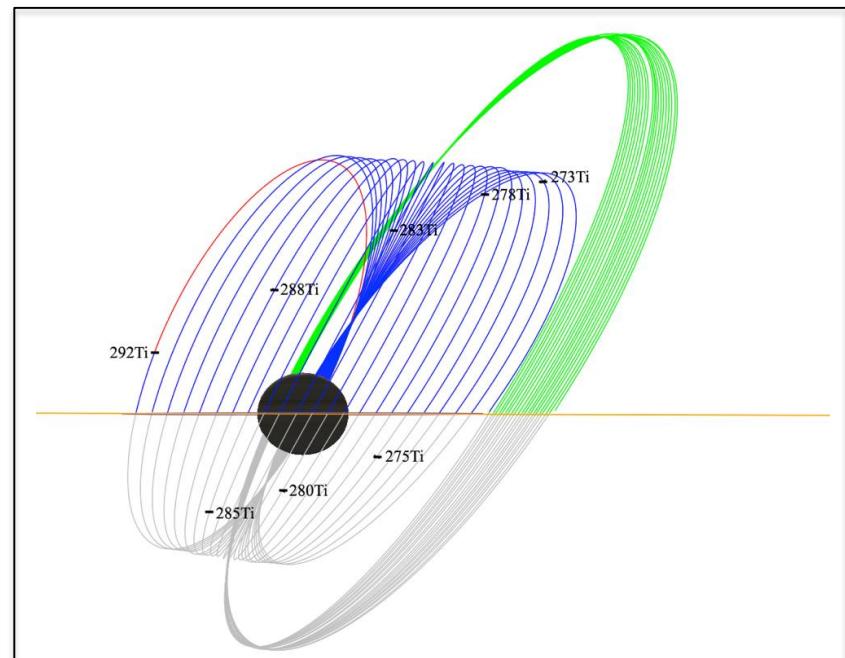
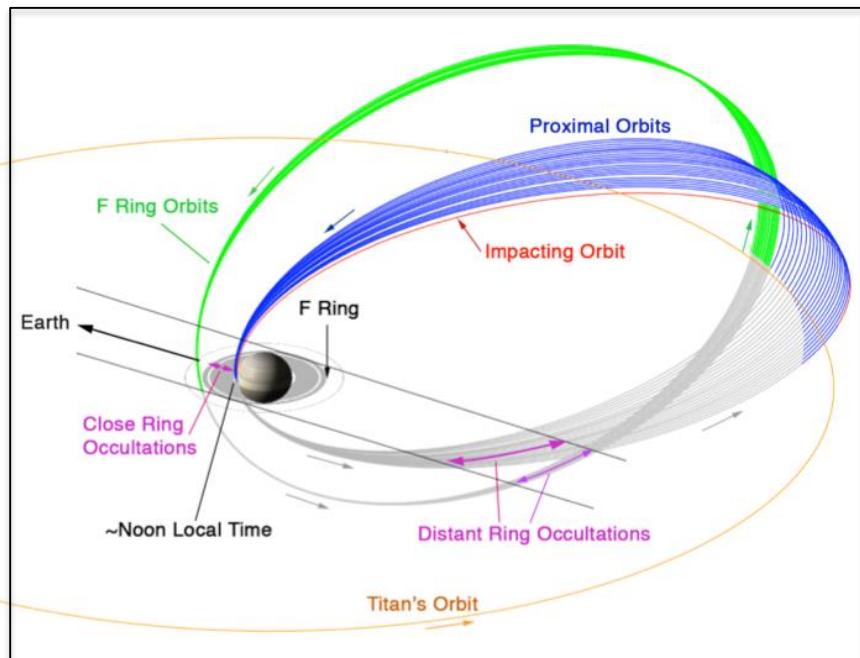
Yungsun Hahn, Duane Roth, and Mau Wong



Jet Propulsion Laboratory
California Institute of Technology

Motivation

Cassini's Grand Finale



F-ring Orbit Grand Finale Orbit Final Impact Orbit
culminating with Saturn atmospheric entry on September 15, 2017

22 stable, highly inclined (62°), short period orbits prior to Saturn impact
Ballistic in nature – theoretically, no maneuvers required after T126

The Problem

Controlling the trajectory within 250 km from the reference path at all times

Why?

- Eliminates late sequence updates: non-standard process, compressed schedule to complete work

How?

- Insert orbit trim maneuvers within proximal mission time span to meet objectives

Science Requirements

- Achieve Saturn atmospheric entry at EOM ($R < 60848$ km, Altitude < 580 km)
- Ensure Cassini is safe from ring particle impact at descending ring plane crossing
- Ensure Cassini is above tumble density at Saturn periapsis ($R > 61750$ km)

Science Requests

- Stay close to reference trajectory overall instead of focusing on flyby target
- Remain within 250 km (68% probability) of reference trajectory
- Provide dispersion plots to assist with science planning
- Do not schedule maneuvers during occultations or within 12 hrs of Saturn periapsis
- Limit number of maneuvers to allow maximum time for science data collection

DESIGN

1. Validation of linear analysis
2. Uncontrolled proximal trajectory
3. Feasible control strategy
 - target placement
 - global search via exhaustive runs
4. Refined control strategy
 - maneuver placement
 - state transition matrix
5. Nominal and backup maneuver strategies

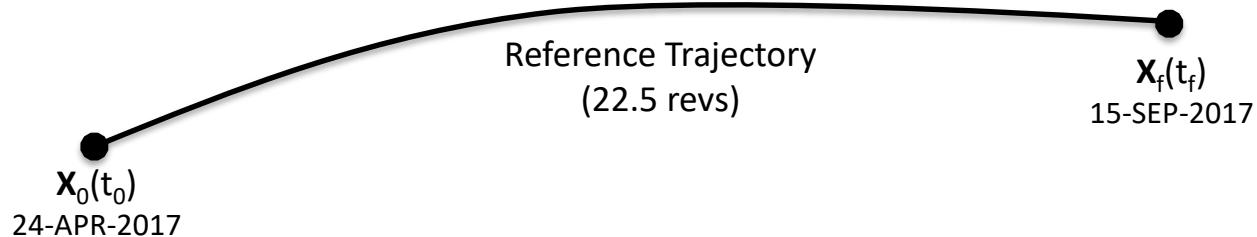
VALIDATION

1. Numerical model readily available for testing purposes
2. Results from the nonlinear analysis already published

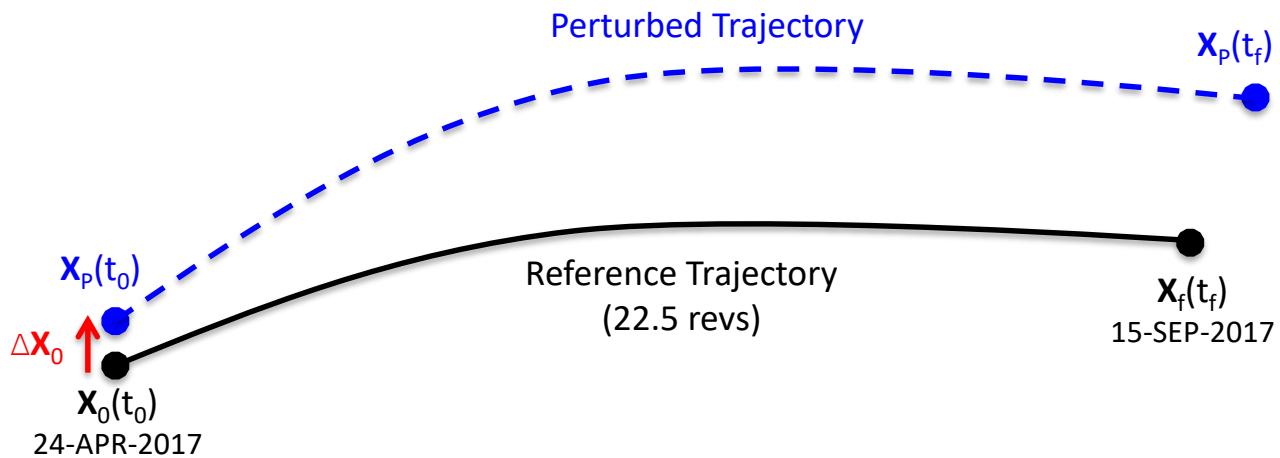
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Validation of Linear Analysis

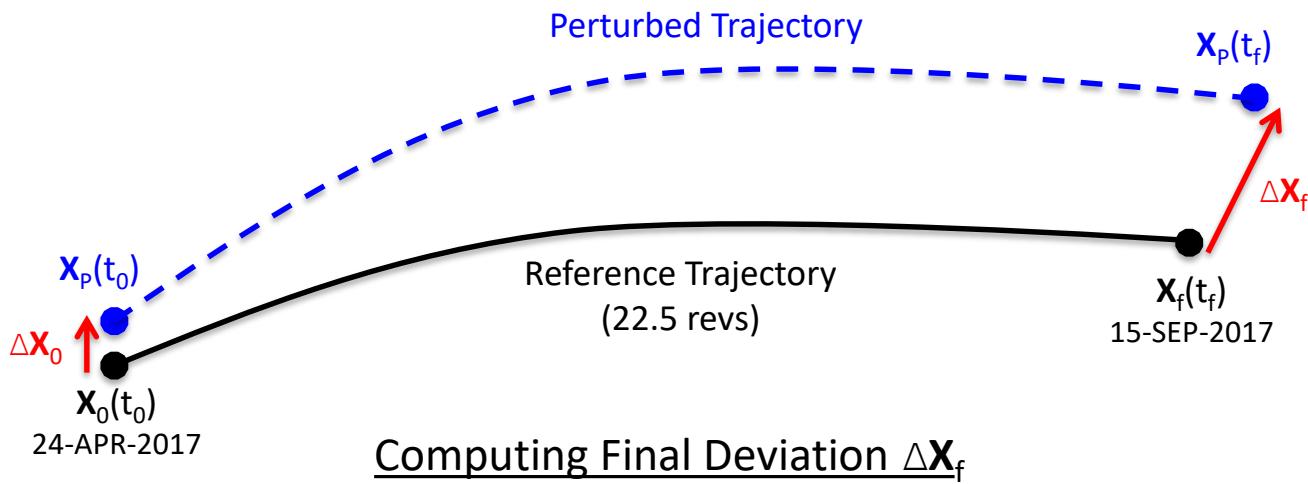
Is the trajectory linear in nature?



Validation of Linear Analysis



Validation of Linear Analysis



1) Linear Mapping via State Transition Matrix (STM)

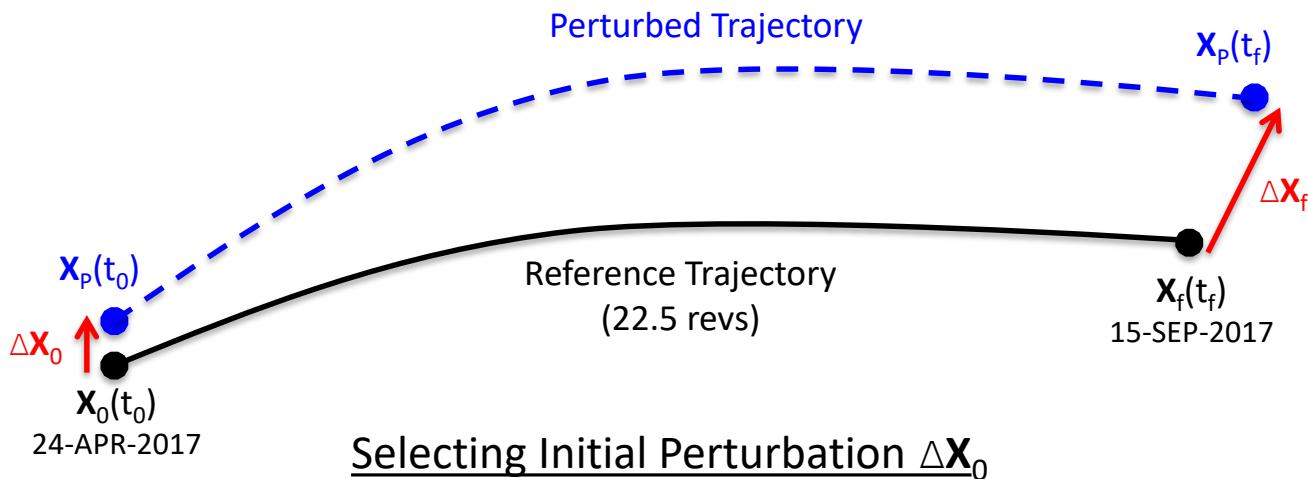
$$\Delta X_f = \Phi(t_f, t_0) \Delta X_0$$

2) Numerical Integration of Perturbed State

Integrate $X_p(t_0)$ from t_0 to t_f

Calculate $\Delta X_f = X_p - X_f$

Validation of Linear Analysis



- 1) From initial 6x6 covariance matrix, calculate eigenvalues (λ_i) and eigenvectors (v_i)

$$SVD = n_1 \lambda_1 v_1 + n_2 \lambda_2 v_2 + n_3 \lambda_3 v_3 + n_4 \lambda_4 v_4 + n_5 \lambda_5 v_5 + n_6 \lambda_6 v_6$$

- 2) Consider six perturbed initial states in the direction of each eigenvector ($n_i = 1$ for $1-\sigma$ perturbation)

$$\begin{aligned}X_{p1} &= X_0 + \sqrt{\lambda_1} v_1 \\X_{p2} &= X_0 + \sqrt{\lambda_2} v_2 \\X_{p3} &= X_0 + \sqrt{\lambda_3} v_3 \\X_{p4} &= X_0 + \sqrt{\lambda_4} v_4 \\X_{p5} &= X_0 + \sqrt{\lambda_5} v_5 \\X_{p6} &= X_0 + \sqrt{\lambda_6} v_6\end{aligned}$$

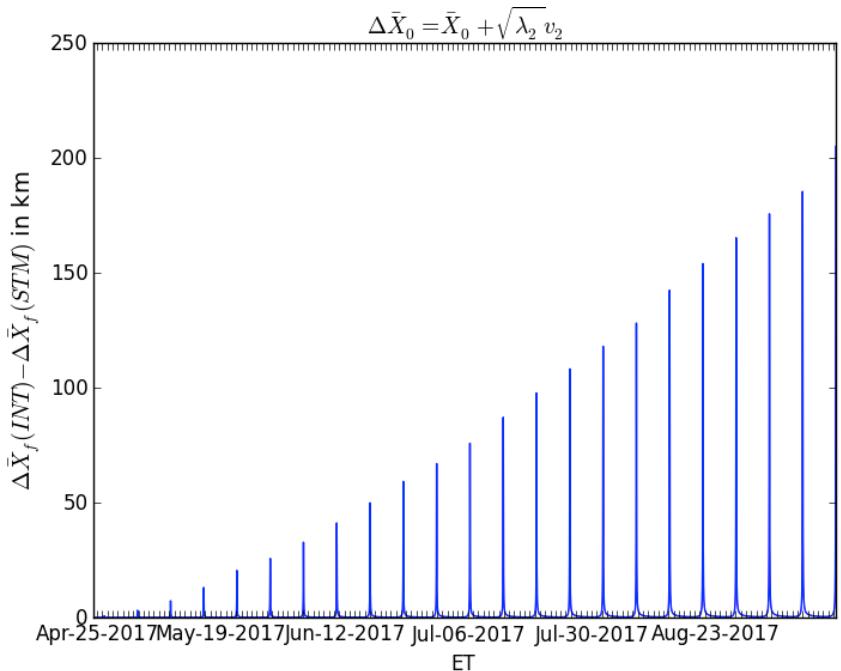
Validation of Linear Analysis

Selected Initial Perturbations: ~ 100 km in position and ~ 1 m/s in velocity

Linear vs. Nonlinear Comparison

- Calculate $\Delta \mathbf{X}_f$ via 1) numerical integration and 2) linear mapping (STM)
- Compute vector difference: $||\Delta \mathbf{X}_{f(INT)} - \Delta \mathbf{X}_{f(STM)}||$
- Plot magnitude of vector difference as a function of time

Worst of all six cases considered
shows a maximum deviation of 200 km



DESIGN

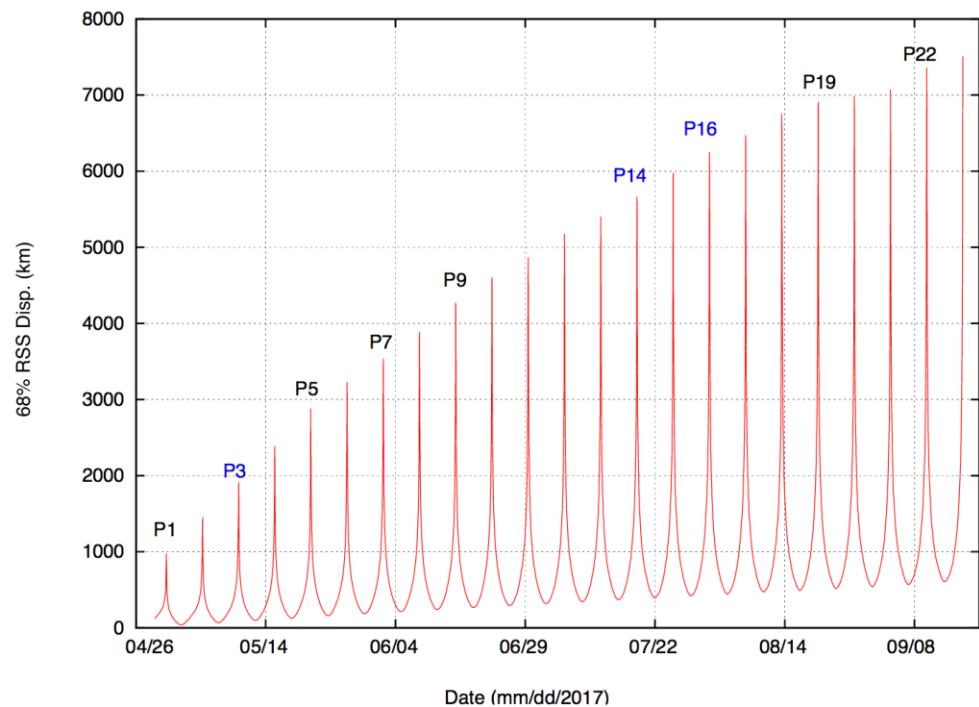
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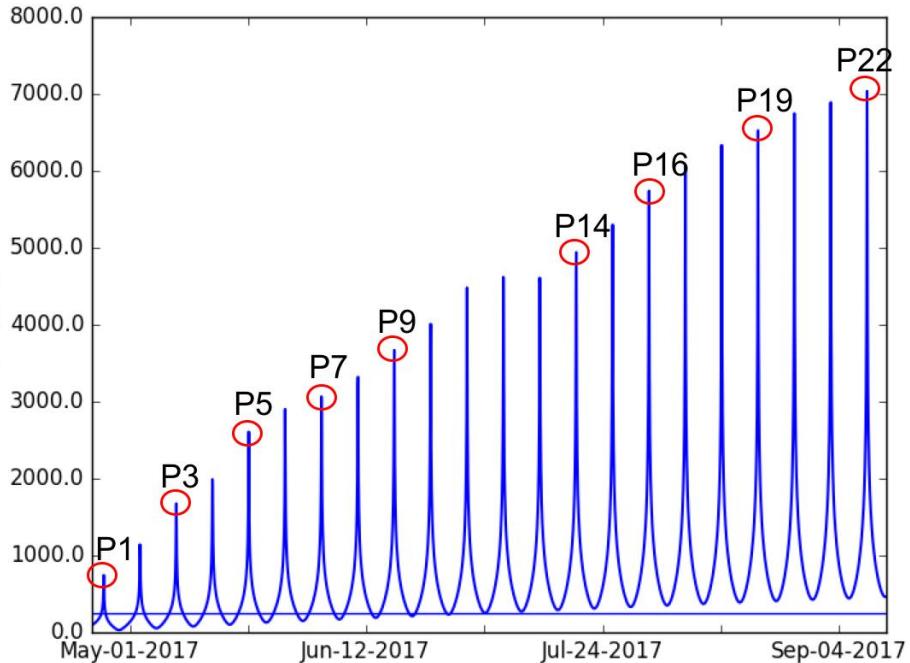
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Uncontrolled Trajectory



Uncontrolled dispersions for the 22 proximal orbits (periapsis-1 through periapsis-22) resulting from the **nonlinear analysis** relying on **numerical integration**.



Uncontrolled dispersions resulting from the **linear analysis** relying on **linear mappings**. The solid blue represents the 250 km control threshold

Peaks / troughs correspond to locations of periapsis / apoapsis.
Solid blue represents the 250 km control threshold

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Objectives

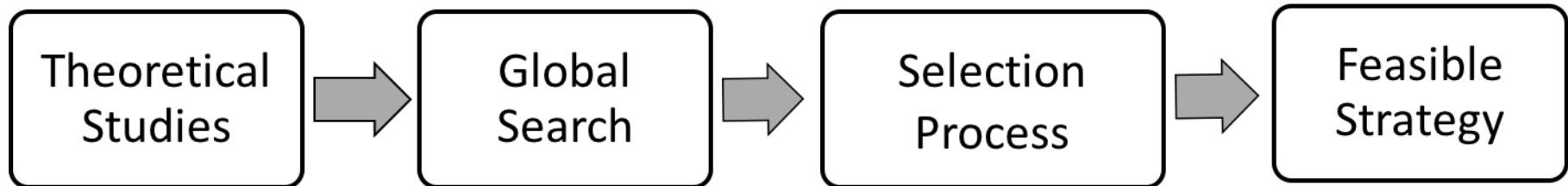
- Determine the **number and location of OTMs** necessary to meet requirements
- Determine the **location of targets**
- Limit propellant consumption

Constraints

- Location of first maneuver is fixed on 24-APR-2017 19:15:00 ET (~2 days after T126)
- Maneuvers must *not* be placed ± 1 day from periapsis
- Backup maneuver opportunity must be included

Scenarios

- One-maneuver strategy
- Two-maneuver strategy
- Three-maneuver strategy



Global Search

Search

- Target location limited to periapsis
- First maneuver location fixed on 24-APR-2017 19:15:00 ET
- All subsequent maneuvers placed +1 day after targeted periapsis
- All possible target combinations considered (1-2-3, 1-2-4, ... , 20-21-22)
- Each combination is run in LAMBIC to collect maneuver and dispersion statistics

Selection

- Parameters:
 - total ΔV ($\mu, \sigma, 99\%$)
 - Average periapse dispersions through 22 revs ($1-\sigma$)
 - Number of periapsis out of bounds
- Requirements:
 - Average periapse dispersions ≤ 250 km
 - Total ΔV (99%) ≤ 2 m/s
 - Number of out-of-bounds periapsis ≤ 5

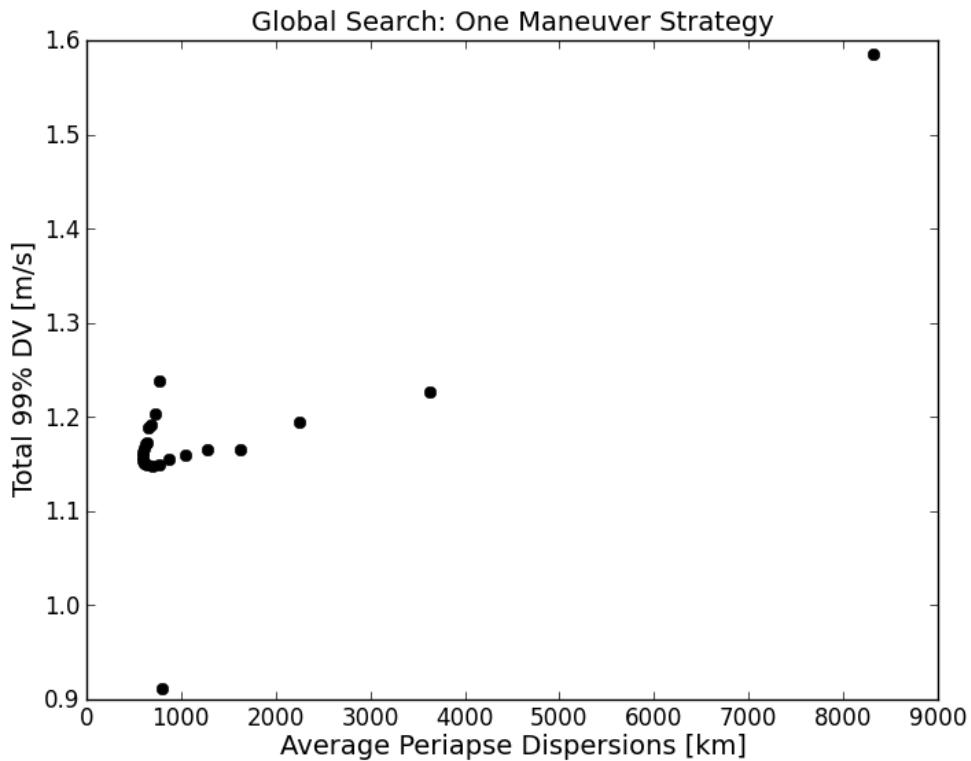
Scenarios

- One, two, three maneuver strategies: 22, 231, 1540 LAMBIC runs

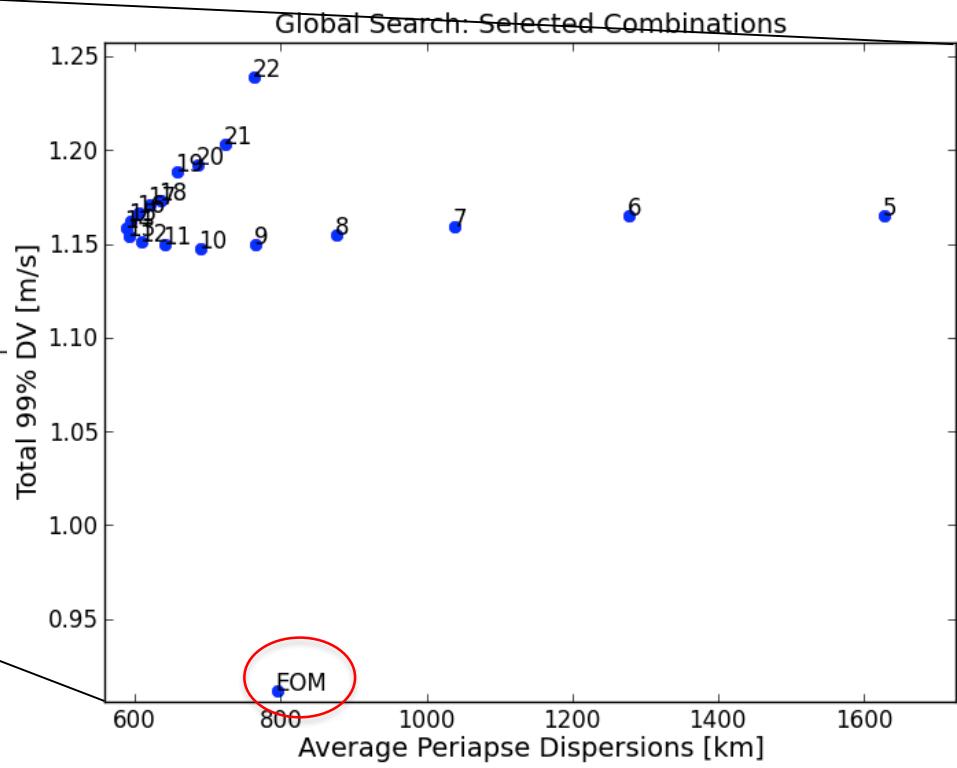
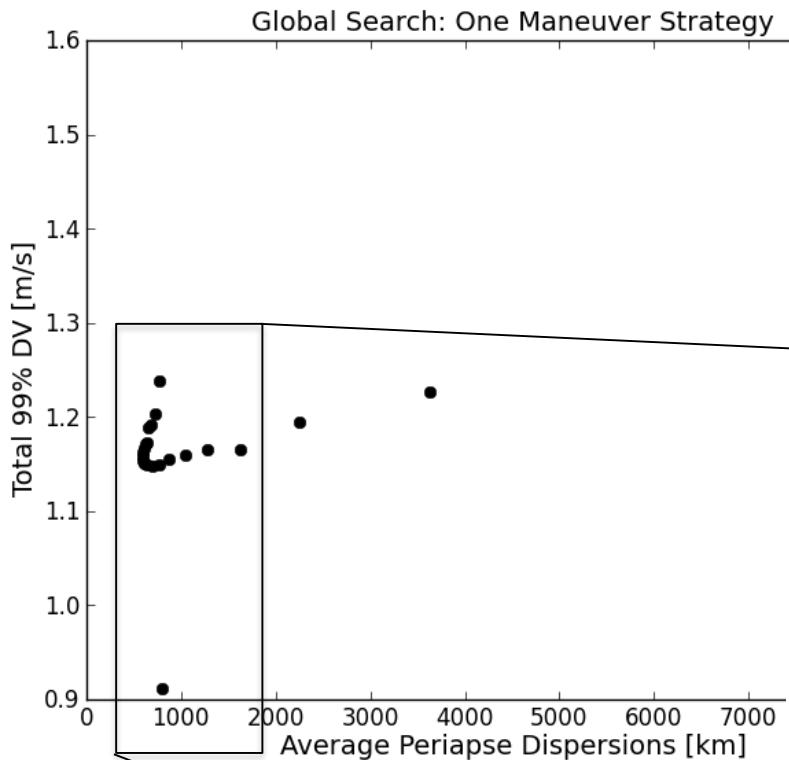
Outcome

- Optimal target location based on brut-force approach

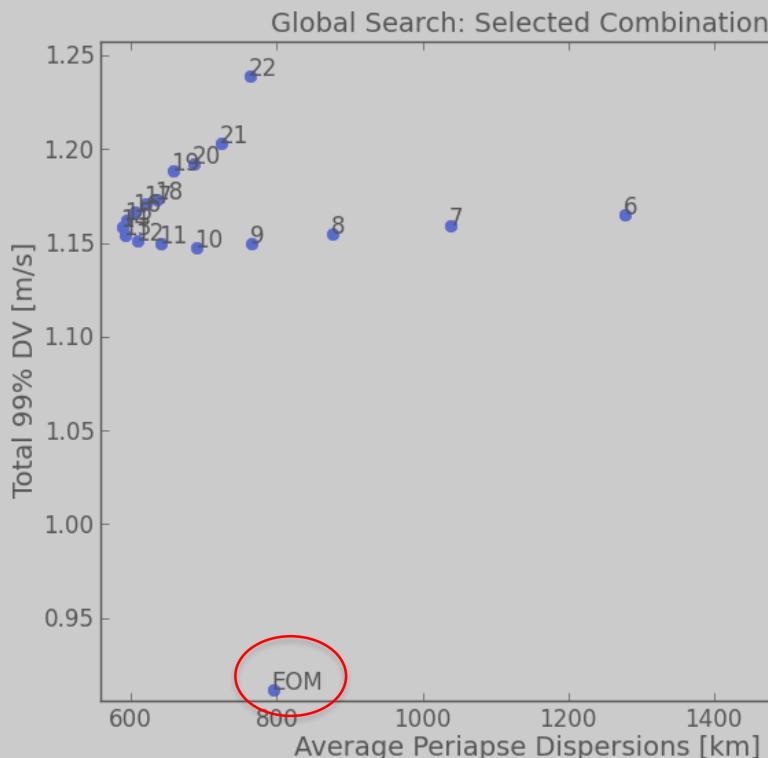
One-Maneuver Strategy



One-Maneuver Strategy

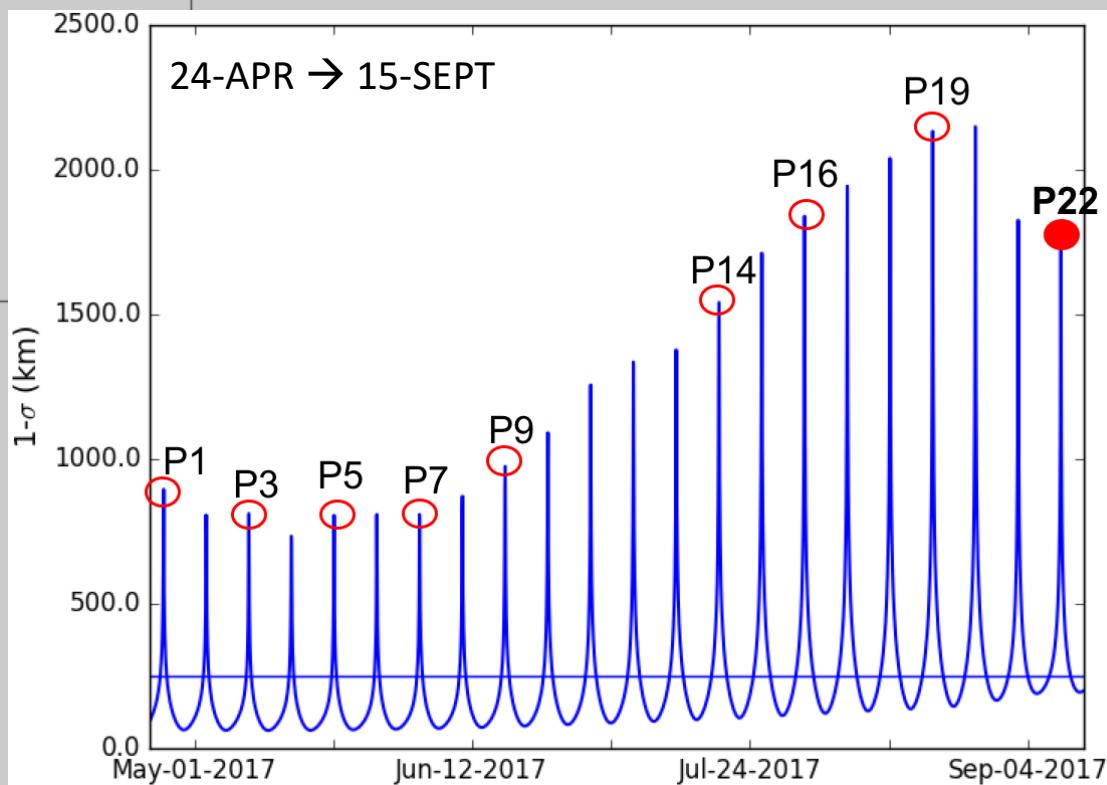


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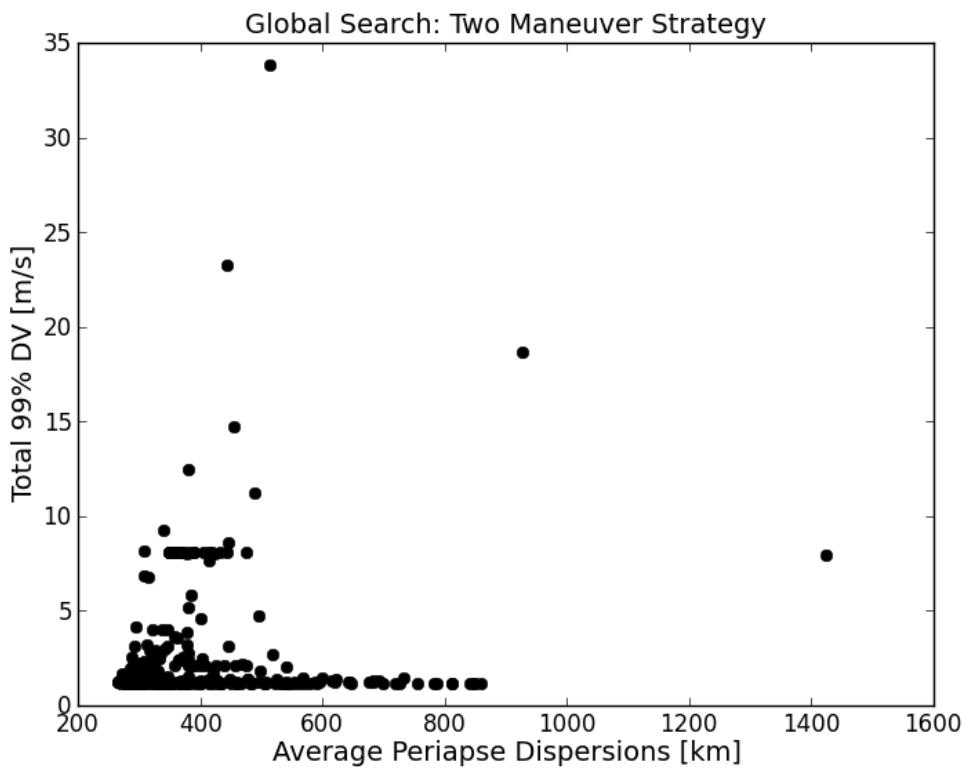


Statistics

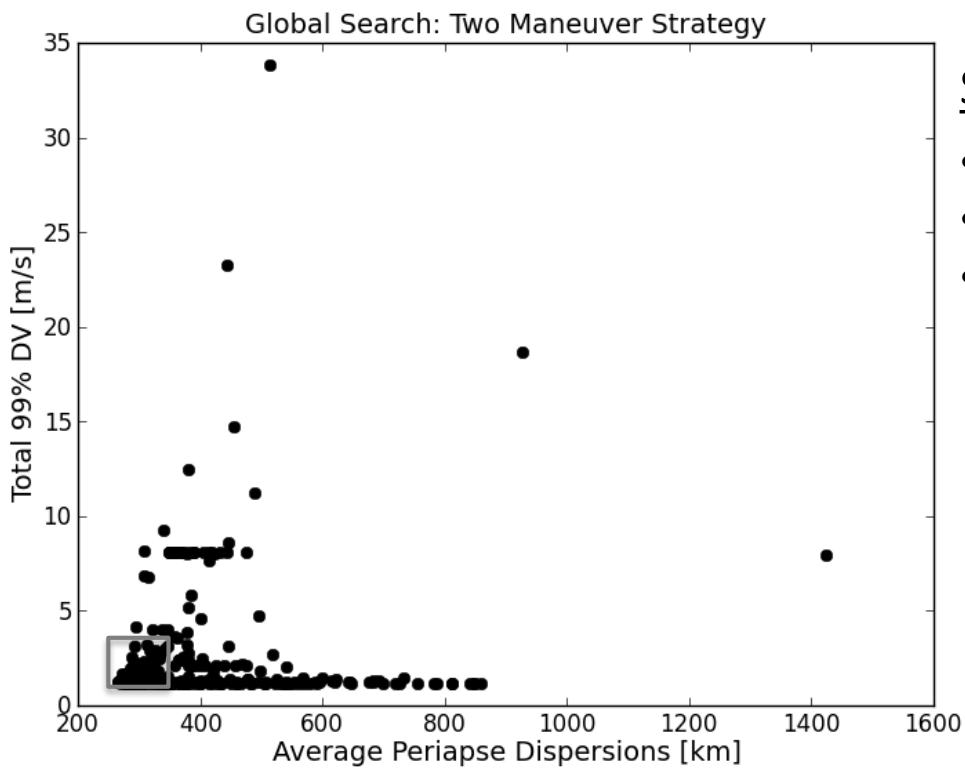
ΔV (mean)	0.398 m/s
ΔV (1-sigma)	0.193 m/s
ΔV (99%)	0.911 m/s



Two-Maneuver Strategy



Two-Maneuver Strategy



Selection Criteria

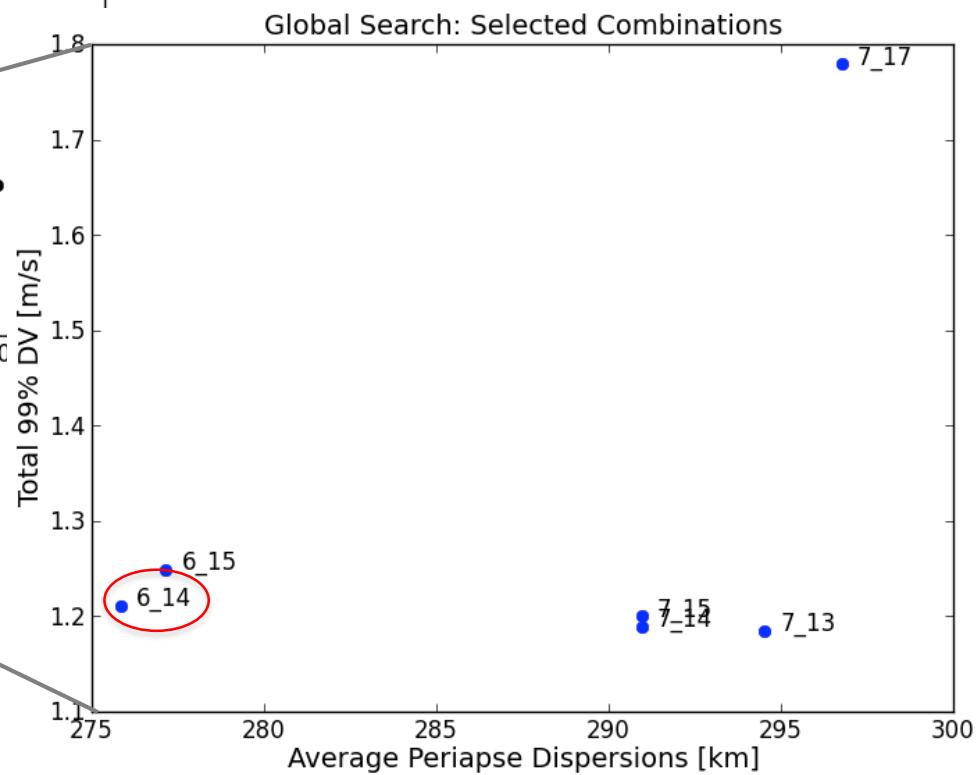
- Average periapse dispersions ≤ 300 km
 - ΔV 99% ≤ 2.0 m /s
 - # of periapsis out-of-bounds ≤ 10

Two-Maneuver Strategy

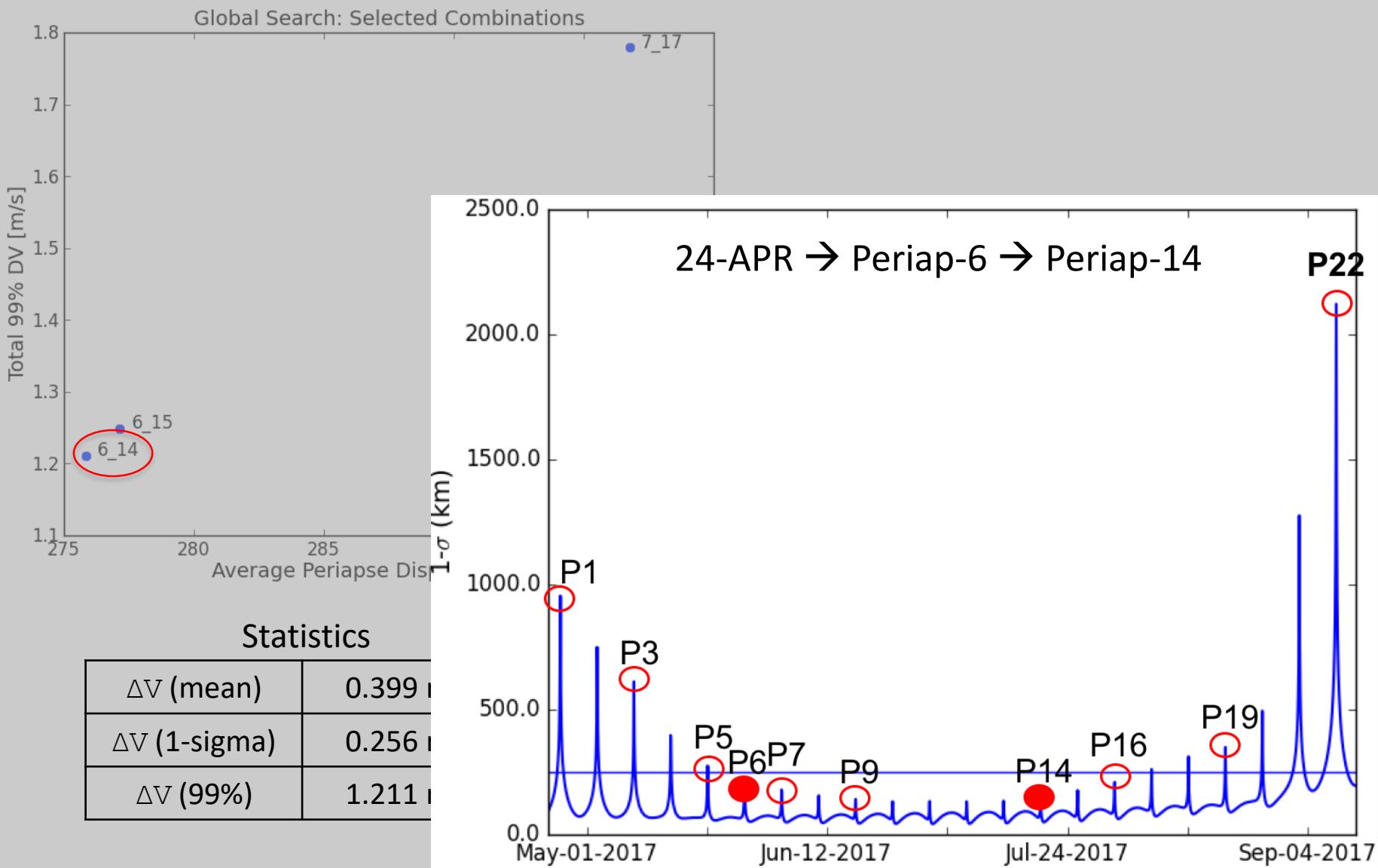


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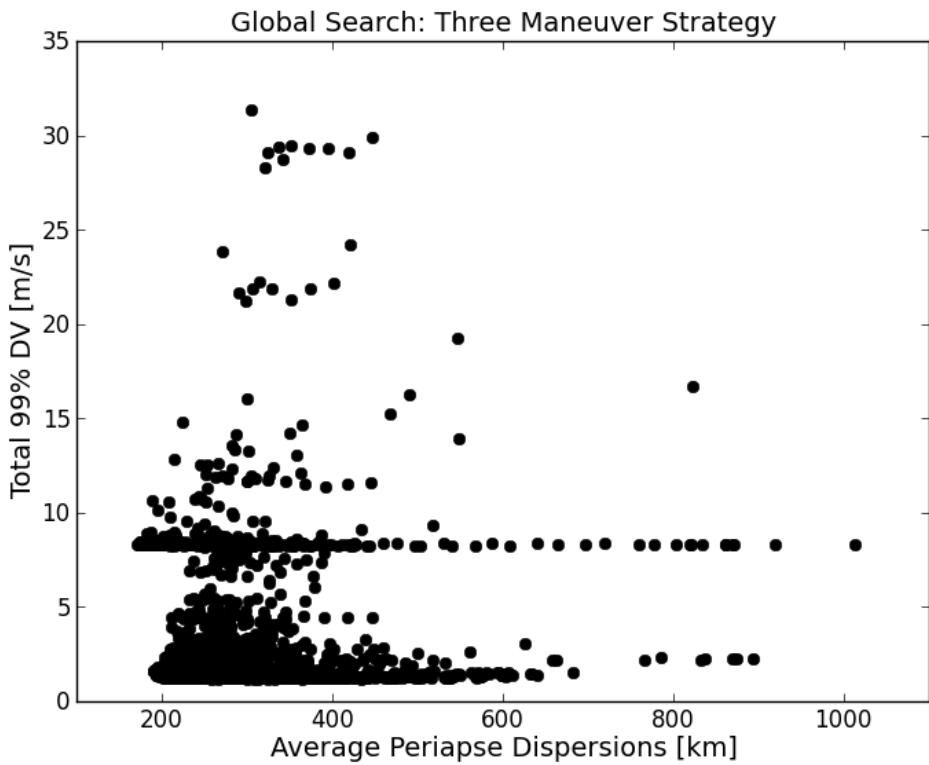
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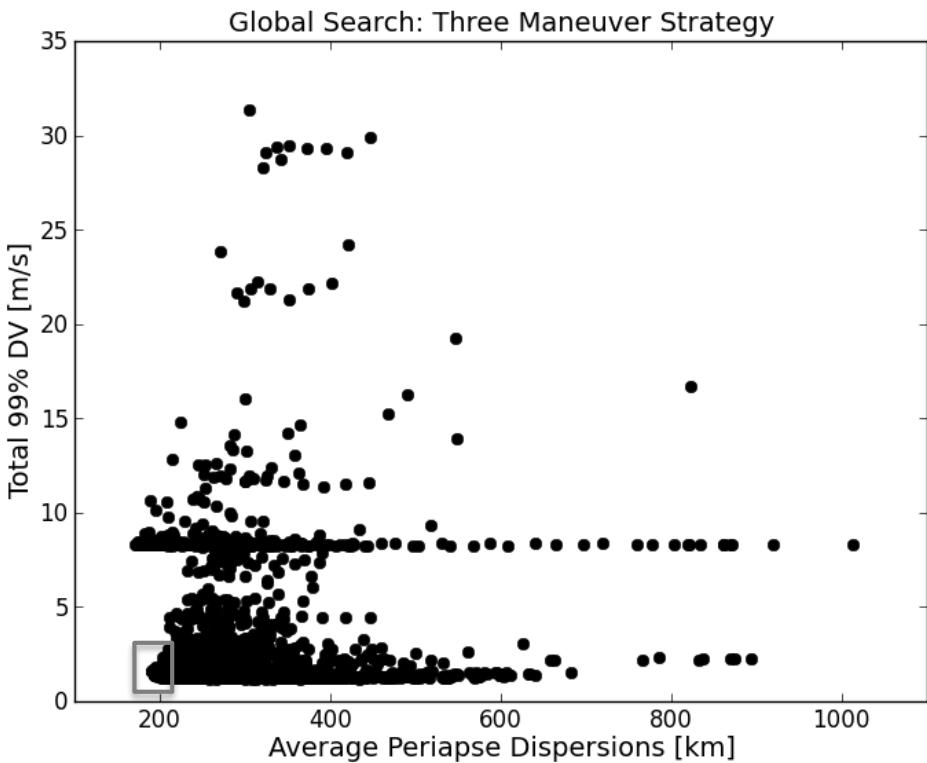
Two-Maneuver Strategy



Three-Maneuver Strategy



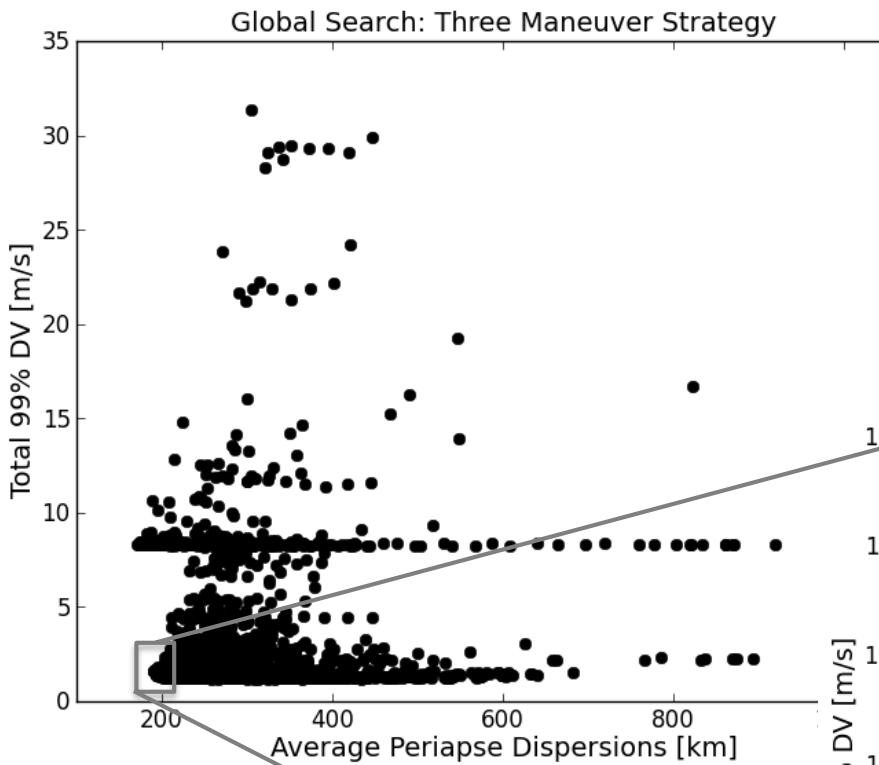
Three-Maneuver Strategy



Selection Criteria

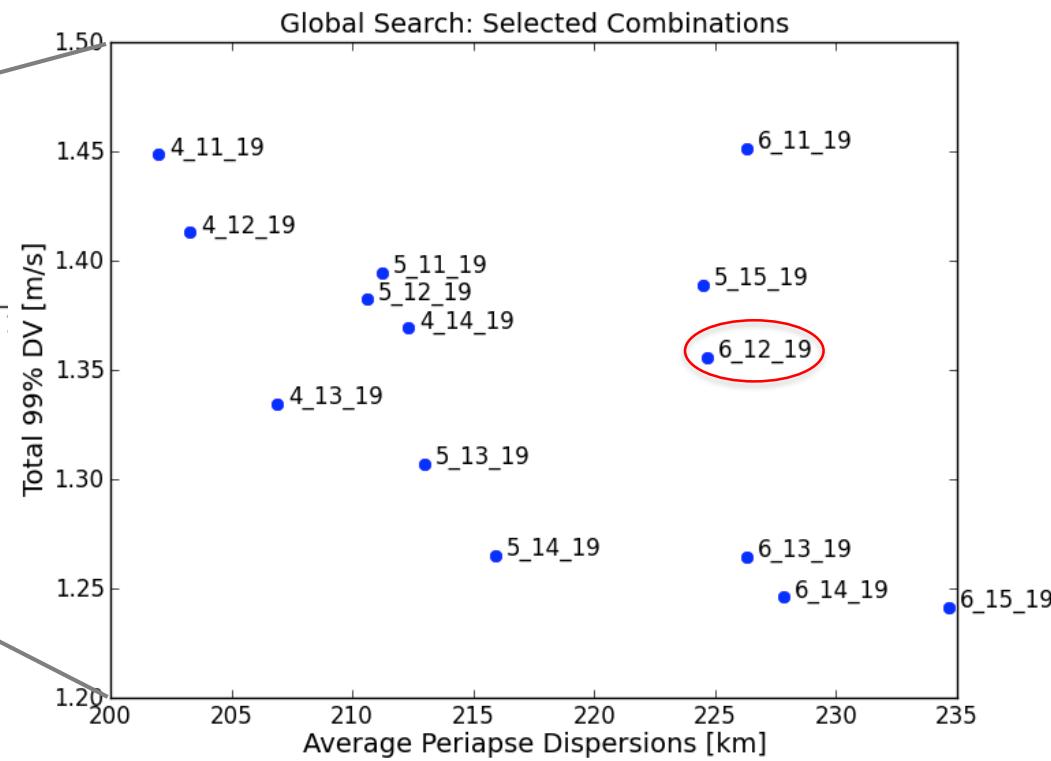
- Average periapse dispersions ≤ 250 km
- ΔV 99% ≤ 1.5 m /s
- # of periapsis out-of-bounds ≤ 5
- Minimize end-dispersions

Three-Maneuver Strategy

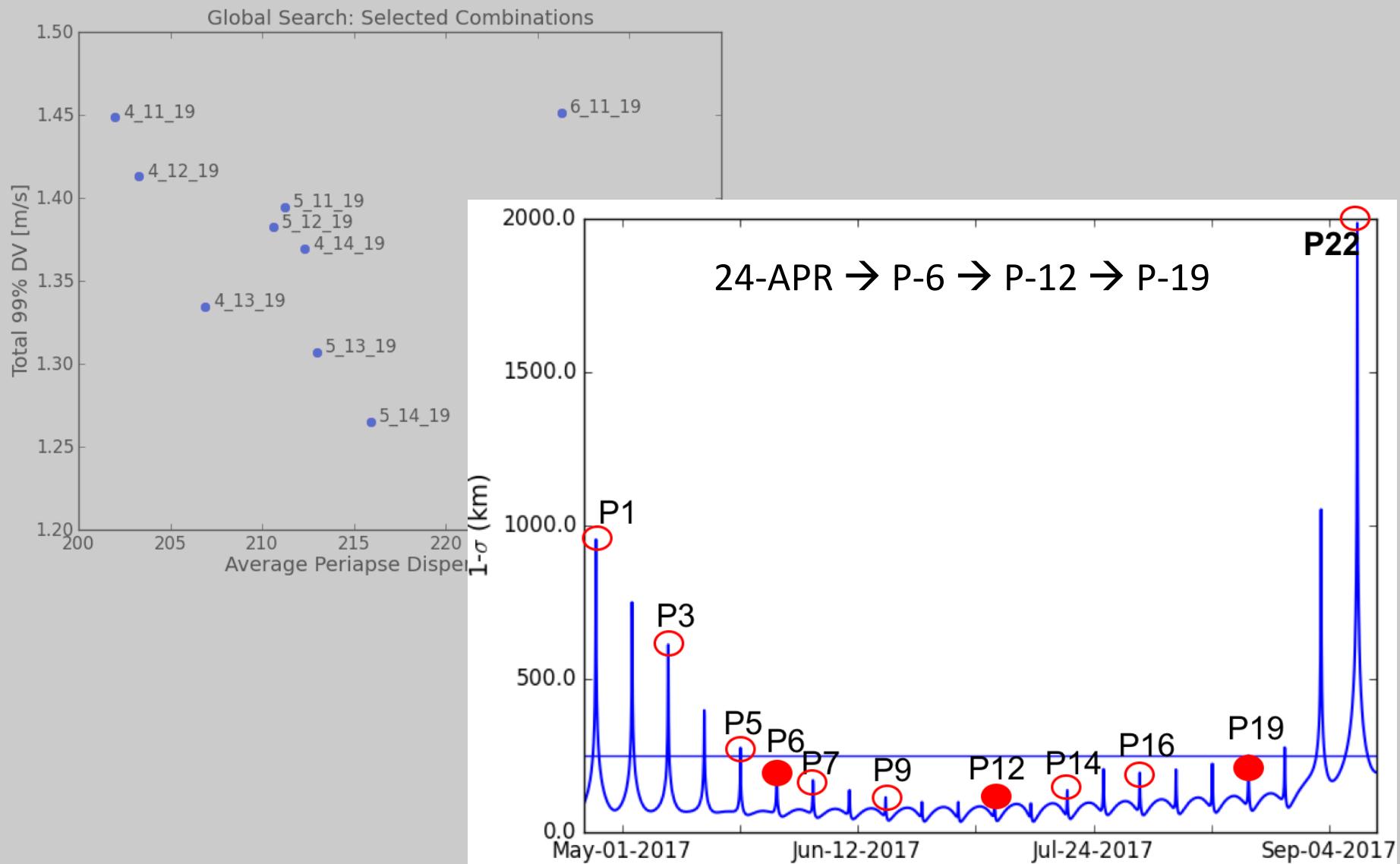


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Three-Maneuver Strategy



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Maneuver Placement



Given a fixed target, state transition matrix (STM) used to determine maneuver location

$$D\bar{V} = K^{-1} D\bar{X}_f$$

$$K = \begin{bmatrix} f_{14} & f_{15} & f_{16} & \frac{\dot{x}_f}{\dot{x}_0} & \frac{\dot{x}_f}{\dot{y}_0} & \frac{\dot{x}_f}{\dot{z}_0} \\ f_{24} & f_{25} & f_{26} & \frac{\dot{y}_f}{\dot{x}_0} & \frac{\dot{y}_f}{\dot{y}_0} & \frac{\dot{y}_f}{\dot{z}_0} \\ f_{34} & f_{35} & f_{36} & \frac{\dot{z}_f}{\dot{x}_0} & \frac{\dot{z}_f}{\dot{y}_0} & \frac{\dot{z}_f}{\dot{z}_0} \end{bmatrix}$$

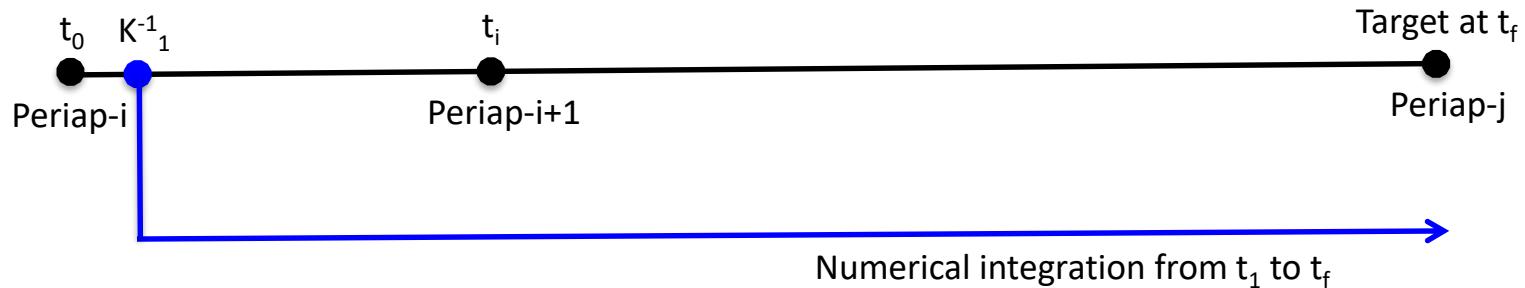


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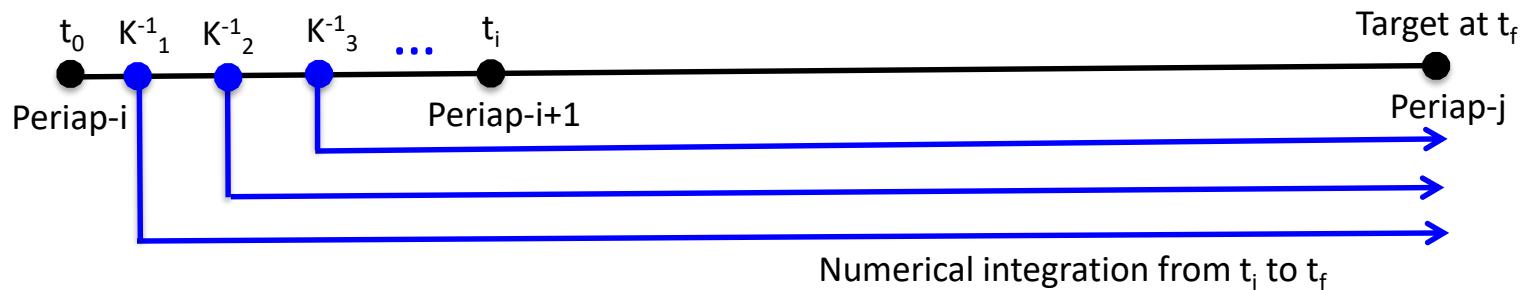
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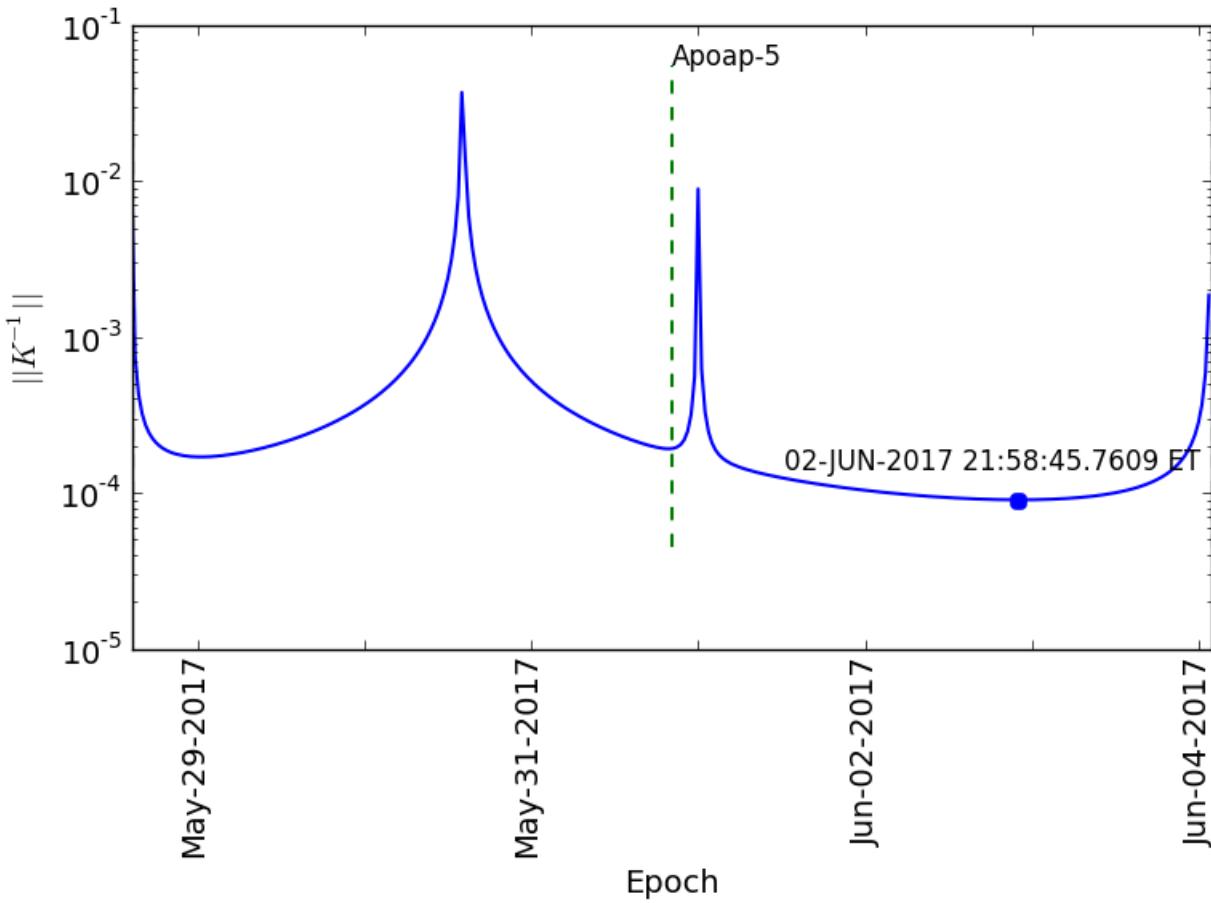
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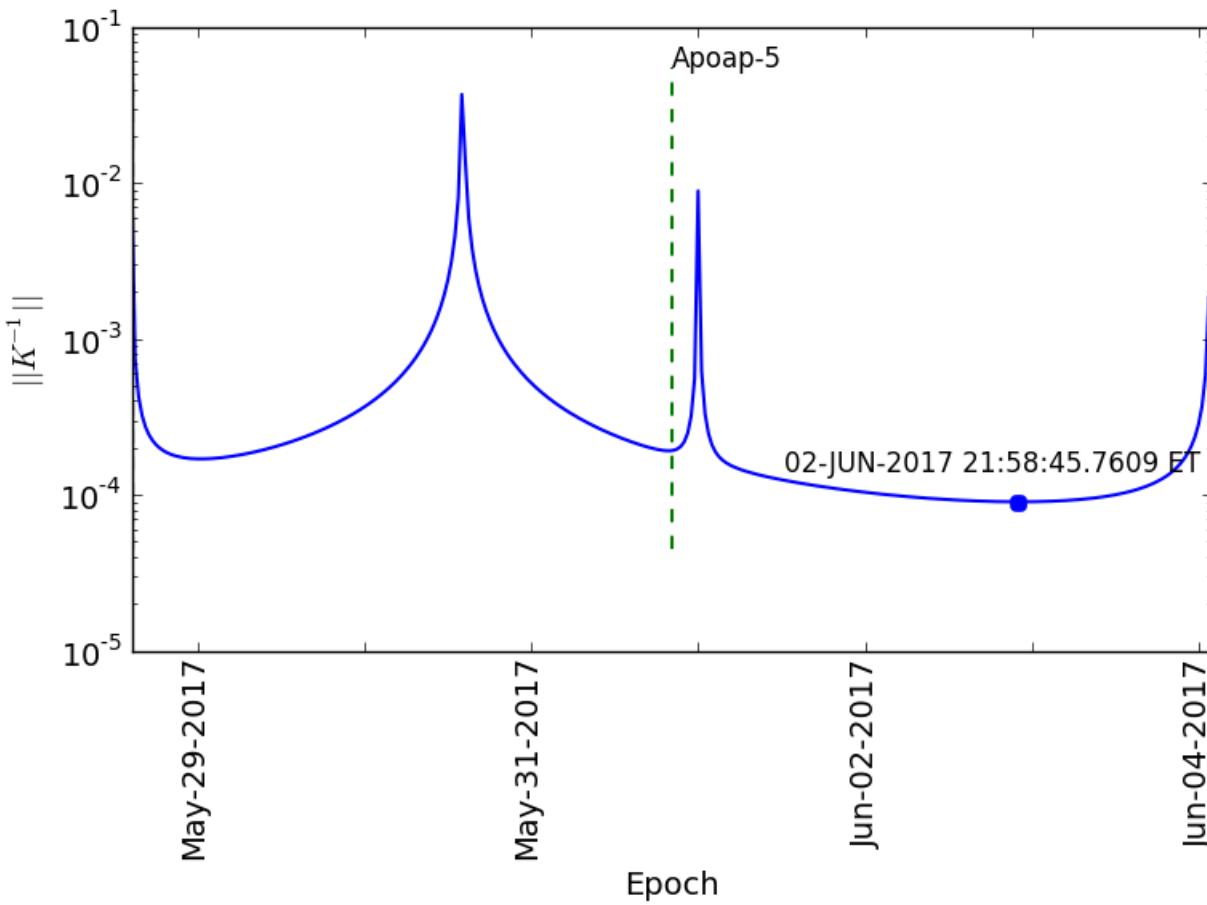
Maneuver Placement Example

Maneuver to be placed between periap-6 (28-May) and periap-7 (4-June)
Target fixed at periap-12 (6-July)



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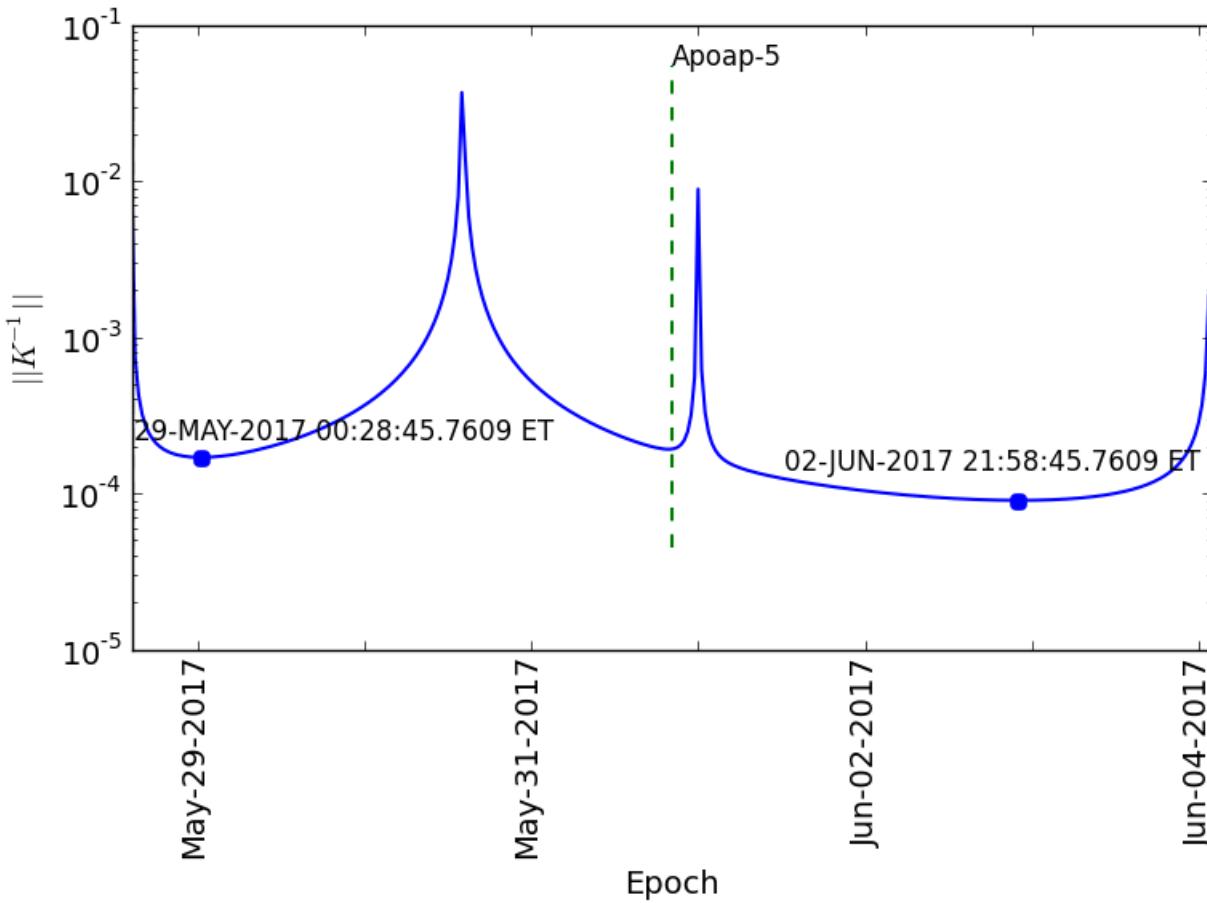
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Not enough time for backup maneuver with selected maneuver location

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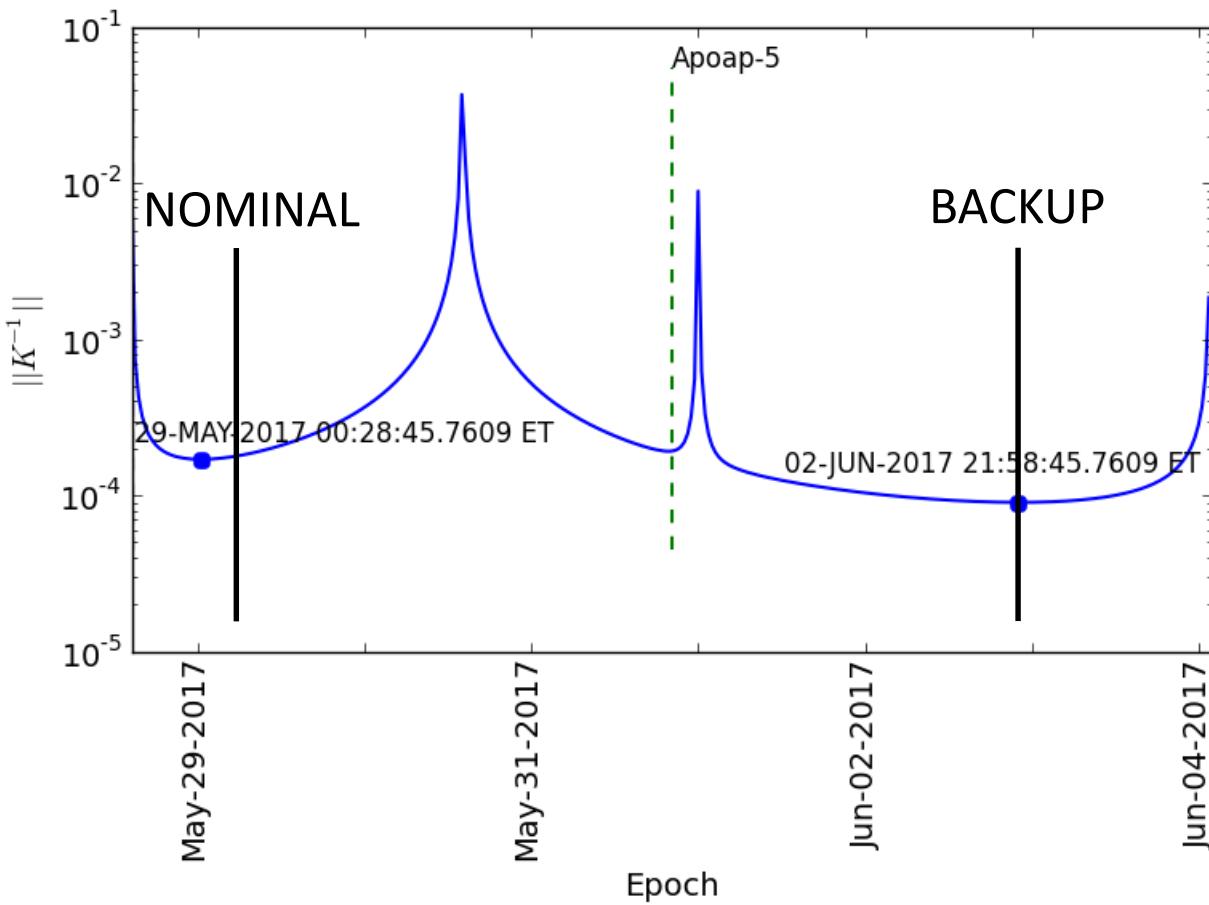


Not enough time for backup maneuver with selected maneuver location

Alternative: set selected location as backup opportunity and set nominal location at next min

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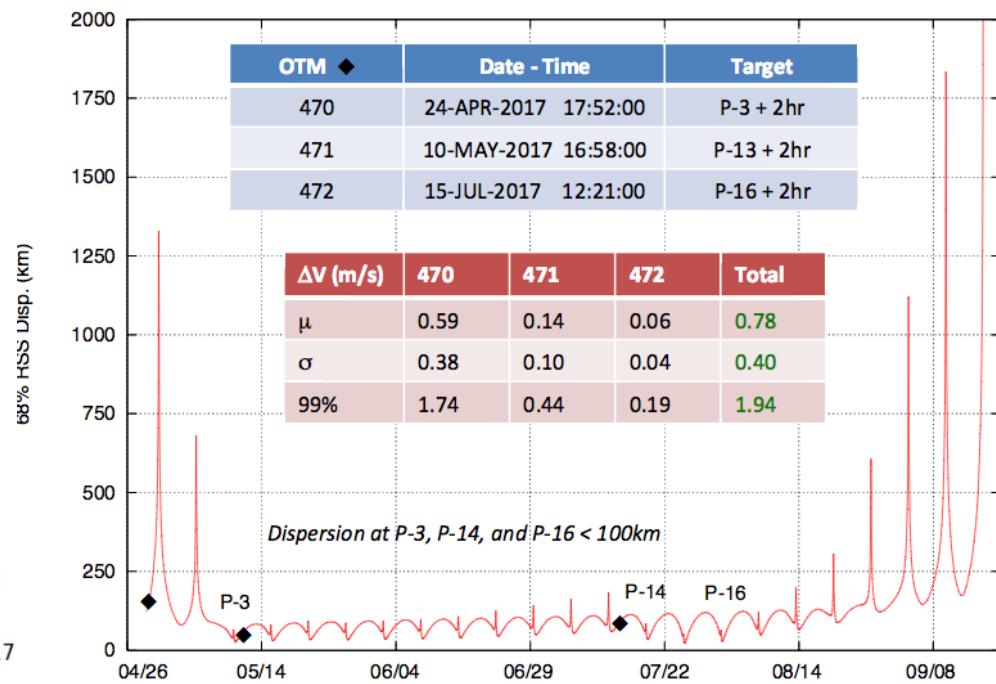
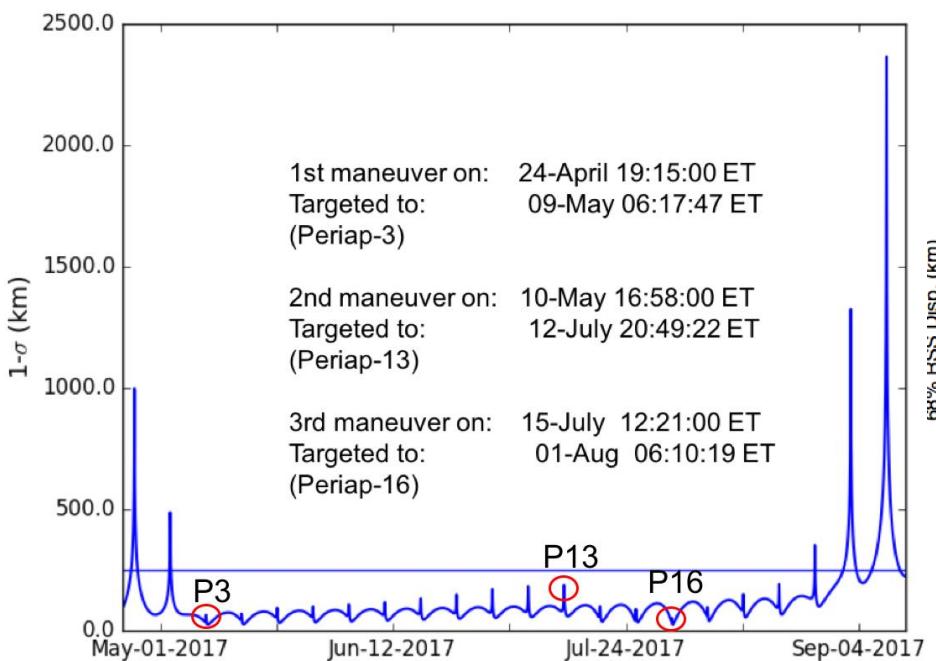
Not enough time for backup maneuver with selected maneuver location

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Final Control Strategy



For reasons related to science observations and sensitivities to timing errors, the Cassini Project decided that there were only three periapses that needed to be controlled and maintained under 250 km: **P3, P13, P16**



ΔV	OTM470	OTM471	OTM472	Total
μ , m/s	0.353	0.032	0.008	0.394
$1-\sigma$, m/s	0.231	0.021	0.011	0.244
99%, m/s	1.119	0.092	0.058	1.193

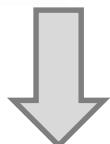
By moving the first target to an earlier time, the position dispersion decreases significantly from over 1300 km at P1 to 60 km at P3, at the expense of increasing the ΔV 99 cost from 0.35 m/s to 0.42 m/s

Summary of Findings



To quickly strategize maneuver and target placement and computing ΔV and dispersion statistics

OBJECTIVE



1. Global search via efficient linear mapping techniques
2. Simple linear approach to optimize maneuver location

DESIGN PROCESS



Evaluate Multiple Strategies

1. Broad view of global solution space
2. Feasible solutions that satisfy all constraints

Short Computational Time

APPLICATION TO CASSINI MISSION



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